## Mark schemes

1. (a) annotates Figure $\mathbf{1}$ to identify equilibrium position; some (or all) of the mark should be below the bob
or
(bottom of mark) should be level with bottom of bob $1 \checkmark$
this is where (the pendulum / bob) is moving fastest / (pendulum has) maximum kinetic energy
or
this is where the transit time is least $2 \sqrt{ }$
for $1 \checkmark$ condone a poorly-annotated sketch if intention is clarified in $2 \sqrt{ }$; do not allow talkout
do not insist on seeing the outline of the card as long as the vertical line is seen;
condone arrows $\uparrow \uparrow$ etc; blobs $\bullet+\mathbf{x}$ are neutral
allow vertical line of the mark to be aligned with either edge of the bob in the left-hand view or marked directly below point of suspension (within one-quarter of bob radius) in the right-hand view, eg

if marks are shown on each view of the pendulum, then each separately must satisfy the criteria for $1 \checkmark$
$2 \checkmark$ is contingent on award of $1 \checkmark$
for $2 \sqrt{ }$ comments about why the mark is not aligned with bob in right-hand view are neutral (at equilibrium) 'acceleration is zero' is neutral
(b) use of appropriate horizontal scale or wtte $1 \checkmark$
use of set-square with edge made vertical or other suitable equipment to eliminate parallax error in $A_{\mathrm{R}} 2 \sqrt{ }$
measures $A_{\mathrm{R}}$ from (either) edge of displaced bob $3 \checkmark$
any of $1 \sqrt{ } 2 \sqrt{ }$ or $3 \checkmark$ can be earned by suitable annotation to Figure 2
for $1 \checkmark$ ruler or 'mm scale' only;
'measuring with a ruler between points marked on the floor' is acceptable (see A below) or use of a 'ruler placed on floor' (see B)

for $2 \sqrt{ }$ allow use of plumb line, spirit level, video or photographic equipment; reject clamp stand
any use of the fiducial mark or the idea that the supporting beam is horizontal are neutral
withhold $3 \sqrt{ }$ unless candidates explains that allowance is being made for radius / diameter of bob (see C and $D$ below)


C
D
MAX 2
(c) extrapolation of curve to (at least) $x=0.70 \mathrm{~m} 1 \checkmark$
for $1 \checkmark$ extrapolation must be continuous and smooth; allow (ruled) straight line;
reject hairy, thick or dashed lines
consistently-recorded min 3 dp values for $T_{0.35}$ and $T_{0.70} 2 \sqrt{ }$
for $2 \sqrt{ }$ allow values seen in working;
$T_{0.35}$ must round to 2.322; condone $T_{0.70}$ by eye
evidence of valid calculation (check denominator correct);
percentage increase in range $1.4(0) \%$ to $1.8(0) \% 3 \checkmark$
don't insist on horizontal or vertical lines between curve and vertical axis on Figure 3
for $3 \sqrt{ }$ expected answer is $1.51 \%$
(d) rejects anomalous 0.247;
average $A_{5} \geq 3$ sf (that rounds to) 0.221 (s) $1 \checkmark$
correct uncertainty calculation or 0.004 (0) (s) seen $2 \sqrt{ }$
or
does not reject 0.247;
average $A_{5}=$ (rounds to) $0.22 \underline{6}(\mathrm{~s})$;
correct uncertainty calculation or $0.01 \underline{5}$ (s) seen $12 \sqrt{ }$
correct \% uncertainty from $\frac{\text { their half range }}{\text { their average }} \times 100 \geq 2 \mathrm{sf}_{3} \checkmark$
0.221 and $1.8 \%$ on answer lines earn $123 \checkmark \checkmark \checkmark$
$1 \checkmark$ ( 0.247 rejected) full answer 0.2214 (s)
$2 \sqrt{ }$ from half range; can be inferred from working
$3 \sqrt{ }$ if $12 \sqrt{ }$ full answer 1.81 \%; allow 1.8 \% or 1 sf 2 \%
when 0.247 is not rejected
$12 \sqrt{ }$ full answer 0.2257 (s)
$3 \checkmark$ full answer 6.647 \%; allow $6.6 \underline{4}$ / $6.6 \underline{5}$ \% or 2 sf $6.6 / 6.7$ \%
for $3 \checkmark$ allow ECF only if uncertainty is from half range
(e) $\ln \left(A_{4} / \mathrm{m}\right)=-1.492 \checkmark$

CAO 3 dp only
(f) vertical scale with one major (cm) grid square $=0.02$ or 0.025 ;
maximum spacing of values marked on the scale $=5 \mathrm{~cm} 1 \checkmark$
points plotted for $n=2,4,7,10$ and 13;
suitable continuous ruled line of negative gradient from $n=2$ to (at least) $n=13$;
line must pass above $n=4$ and $n=10$ points
and must pass below $n=7$ point $2 \sqrt{ }$

for $1 \checkmark$ scale might go (down page) from -1.40 to -1.72 (1 square $=$ 0.02 ) or from -1.30 to -1.70 (1 square $=0.025$ );
scale must cover range of plotted points; do not insist on use of broken scale convention
no credit for reversed values leading to graph with 'positive' gradient;
no credit for missing / inconsistent minus signs or for inconsistent dp in labelled values
for $2 \sqrt{ }$ allow ECF acceptable line based on accurate plot of incorrect $n=4$ point
allow ECF for graph with 'positive' gradient due to reversed scale, eg line must pass below $n=4$ and $n=10$ points and must pass
above $n=7$ point
accept only 4 points if $n=4$ is not tabulated; line must pass
between $n=7$ and $n=10$
ignore any plot of $\operatorname{In}\left(A_{5} / m\right)$ based on part (d) data;
withhold mark for poor points eg blobs or for thick / faint /
non-continuous line
(g) (any) correct expression with $\operatorname{In} A_{n}$ as subject ${ }_{1} \checkmark$
$\delta=e^{\text {-gradient }}$ or wtte ${ }_{2} \sqrt{ }$
for $1 \sqrt{ } y=m x+c$ idea is required
either $\ln A_{n}=-n \ln \delta+\ln A_{0}(y=m x+c)$
or $\ln A_{n}=\ln A_{0}-n \ln \delta(y=c+m x)$
not $\ln A_{n}=\ln A_{0}-\ln \delta^{n}$
treat $\lg A$ ' as a slip
allow use of $\log A^{\prime}$ for $1 \checkmark$ but no ECF in $2 \sqrt{ }$
for $2 \sqrt{ }$
$\delta$ must be the subject, reject In $\delta=-$ gradient etc
allow ECF if $1 \checkmark$ is withheld for missing - sign;
if gradient is evaluated accept $\delta=e^{(+) 0.024}$ or $\delta=1.02(4)$ etc an explanation that $\delta$ can be found using $A_{n}=A_{0} \delta^{-n}$ must rely on values of $A_{n}, A_{0}$ and $n$ that are determined using the line in

## Figure 4

2. C
3. B
4. (a) Evidence of distance travelled = area under graph $=1755+1440+1620=4815 \mathrm{~V}$ Full marks can be credited for use of suvat.

Average speed $=$ total distance/time taken $=4815 / 240$
$=20.1 \mathrm{~m} \mathrm{~s}^{-1} \checkmark$ (at least 3sf)
Which is less than (speed) limit, (and therefore the answer is No). $\checkmark$
Allow ecf for distance in MP2
Only award MP3 for incorrect speed if attempt made to calculate distance correctly e.g. area under graph OR a.e. in distance or speed

Alternative for MP2 and MP3
Calculation of distance travelled at speed limit $=5280 \mathrm{~m} \checkmark$
Which is greater than distance travelled (so no). $\checkmark$
Alternative for MP1 and MP2:
Total area $=80.25 \mathrm{~m} \mathrm{~s}^{-1} \mathrm{~min} \checkmark$
Time $=4 \mathrm{~min}$
Average $=20.1 \mathrm{~m} \mathrm{~s}^{-1} \checkmark$
(b) Using reaction time of $2.0 \mathrm{~s} \checkmark$

Use of distance $=$ speed $\times$ time $=62 \mathrm{~m}$.
62 m (would be appropriate). $\checkmark$
Award MP2 if 1.6 s (to give 50 m ) or 1.8 s (to give 56 m ) or 1.7 s (to give 53 m ) or average of two distances used
Allow 60 m.
(c) Use of $\mathrm{F}=\mathrm{ma}$ to calculate acceleration.
$a=6800 / 1200 \checkmark=5.7 \mathrm{~m} \mathrm{~s}^{-2}$
evidence of use of suvat to calculate s or $t, \checkmark$
to give $t=5.5 \mathrm{~s} \checkmark$
$s=85 \mathrm{~m} . \checkmark$
If no other mark given, allow 1 mark for
$m v=1200 \times 31$ (= 37200)
Alternative for MP1 and MP2
$t=\frac{m v-m u}{F}$
Allow ce for a.
Allow ce for either incorrects or $t$.
(d) (It is assumed that) the car in front would take the same time/travel the same distance as the car behind when braking/ only difference is reaction time of the driver of car behind. $\checkmark$ Or
Car in front cannot stop instantaneously (so car behind will have time/distance to bring car to rest).or words to that effect

Alternative:
suggestion that total stopping distance is too large (drivers would ignore it/inefficient use of motorway)
(e) Correct use of $\cos (5) \checkmark$

## E.g.

$m g=N \cos (5)$
Correct use of $\sin (5) \checkmark$
E.g.
$N \sin (5)\left(=m v^{2} / r\right)$
So
$m v^{2} / r$ seen $\checkmark$
And $v=(r g \tan (5))^{1 / 2}$
Gives $v=(200 \times 9.81 \times \tan (5))^{1 / 2}=13$
So speed limit $=13 \mathrm{~m} \mathrm{~s}^{-1} \checkmark$
May see cos (85) for sin (5)
Alternative for MP1 and MP2: Evidence of $m g$ tan (5)
fourth mark is for answer and suggesting this as the speed limit.
Max 3 if $\mathrm{mg}=\mathrm{N}$ used
5. D $\frac{2 \pi^{2} m A^{2}}{T^{2}}$

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\frac{2 \pi^{2} m A^{2}}{T^{2}}
$$

6. B

$$
2 T \quad T
$$

$7 .{ }^{\text {B }}$
8. (a) $1.5(\mathrm{~ms}) \checkmark$
(b) $\quad A=4.2(\mathrm{~mm})$ read from graph $\checkmark$
$\mathrm{T}=2.0(\mathrm{~ms})$ read from graph $\checkmark$
$\left(a_{\max }=4.2 \times 10^{-3} \times\left(2 \times \pi /\left(2 \times 10^{-3}\right)\right)^{2}\right.$
$4.1(5) \times 10^{4}\left(\mathrm{~m} \mathrm{~s}^{-2}\right) \checkmark$ (Do not allow 4.2)
Condone power of ten error in $A$ and/or $T$ but not in final answer.
Evidence for $T$ might be seen in equation, as 500 ( $f$ ).
Only allowed ecf for max 2 is use of 4.1 mm for A , giving $4.0 \times 10^{4}$ ( $\mathrm{m} \mathrm{s}^{-2}$ )
(c) longitudinal
(they) oscillate along direction of energy transfer $\checkmark$
Both required for 1 mark
Condone "vibrate" for oscillate.
Condone 'travel' for transfer
9.
10.
11.
(a) SHM is when

The acceleration is proportional to the displacement $\checkmark$
the acceleration is in opposite direction to displacement $\sqrt{ }$
(b) $f=1 / T=1 / 0.05=20 \mathrm{~Hz}$ V
$\left(v_{\max }=2 \pi f A\right)$
$A=\frac{0.044}{2 \pi \times 20} \checkmark\left(=3.5 \times 10^{-4} \mathrm{~m}\right)$
(c) Cosine shape drawn, maximum at $t=0$, amplitude $3.5 \times 10^{-4} \mathrm{~m} \checkmark$
(d) (any of the following when the velocity is zero) $0.00 \mathrm{~s}, 0.025 \mathrm{~s}, 0.050 \mathrm{~s}$ or $0.075 \mathrm{~s} \checkmark$
(e) when the vibrating surface accelerates down with an acceleration less than the acceleration of free fall the sand stays in contact. $\checkmark$
above a particular frequency, the acceleration is greater than $g \checkmark$
there is no contact force on the sand $O \boldsymbol{R}$
sand no longer in contact when downwards acceleration of plate is greater than acceleration of sand due to gravity $\checkmark$
(f) (when the surface acceleration is the same as free fall)
$g=r \varpi^{2}=\mathrm{A}(2 \pi f)^{2} \checkmark$
$f=\sqrt{ }\left(\mathrm{g} / \mathrm{A} 4 \pi^{2}\right)=\left(9.81 /\left(3.5 \times 10^{-4} \times 4 \pi^{2}\right)\right)^{1 / 2}=26.6(7) \mathrm{Hz} \checkmark$
12. C
13. D
14. D
15. A
16. D
17. D
18.

$$
\text { (a) } \begin{aligned}
& \text { (use of } \left.v=2 \pi f \sqrt{a^{2}}-x^{2}\right) \\
& v_{\max }=2 \pi \times 2.0 \times 2.5 \times 10^{-2} \\
& v_{\max }=0.314 \mathrm{~m} \mathrm{~s}^{-1} \checkmark \\
& \left(\text { use of } \mathrm{E}_{\mathrm{k}}=1 / 2 \mathrm{mv}^{2}\right) \\
& 54 \times 10^{-3}=1 / 2 \mathrm{~m} \times(0.314)^{2} \\
& m=1.1(\mathrm{~kg}) \checkmark \\
& f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}} \\
& 2.0 \times 2 \pi=\sqrt{ }(\mathrm{k} / 1.1) \checkmark \\
& \left(k=(4 \pi)^{2} \times 1.1\right) \\
& k=173(172.8) \checkmark\left(\mathrm{N} \mathrm{~m}^{-1}\right) \\
& \quad \operatorname{Can}
\end{aligned}
$$

OR
$5.4 \times 10^{-3}=1 / 2 k\left(2.5 \times 10^{-2}\right)^{2} \checkmark$
$k=173$ (172.8) $\mathrm{N} \mathrm{m}^{-1} \checkmark$
If either of these methods used can then find mass from frequency formula or from kinetic energy

OR
$54 \times 10^{-3}=1 / 2 F \times 2.5 \times 10^{-2}$
$F=4.32$
$4.32=k \times 2.5 \times 10^{-2}$
$k=173\left(\mathrm{~N} \mathrm{~m}^{-1}\right)$
Accept 170 and 172.8 to 174
(b) (use of $f=\frac{1}{2 \pi} \sqrt{\frac{k}{m}}$ ))
same mass so $f \propto \sqrt{k}$
thus frequency $=2.0 \times \sqrt{3}$
frequency $=3.5(3.46)(\mathrm{Hz}) \checkmark$
Allow CE from (a) for $k$ or $m$
(c) Two from:

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(resonance) peak / maximum amplitude is at a higher frequency \checkmark
due to higher spring constant \checkmark
(resonant) peak would be broader \checkmark
due to damping \checkmark
amplitude would be lower (at all frequencies) \checkmark
due to energy losses from the system \checkmark
    First mark in each case for effect
    Second mark for reason
    2 marks max for effects
    2 marks max for reason
    Cannot award from sketch graph unless explained
    First mark in each pair stand alone
    Second mark conditional on first in each pair
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19. D
20. D
